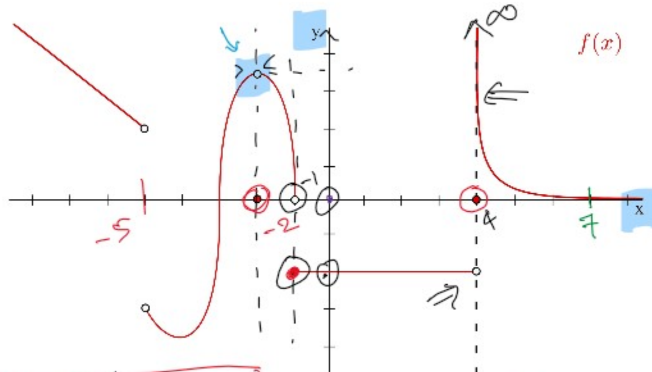
TEXAS A&M UNIVERSITY
College of Arts
& Sciences

MATHEMATICS
Math 142 - Spring 2024
Week in Review 1: 1.1, 1.2

Week-in-Review 1 (1.1, 1.2)

Problem 1. Find the following limits, if they exist, based on the graph of $f(x)$ below:



$\boxed{f(-5) \text{ DNE}}$

(1) $\boxed{\lim_{x\to -6} f(x) = 3}$

$\lim_{x\to -5^-} f(x) = 2$ (LHS)

LHS ≠ RHS

$x=-5$ → (2) $\lim_{x\to -5} f(x)$

$\lim_{x\to -5^+} f(x) = -3$ (RHS)

$\lim_{x\to -5} f(x)$ DNE

(3) $\lim_{x\to -2} f(x)$

LHS : $\lim_{x\to -2^-} f(x) = 3.5$

RHS : $\lim_{x\to -2^+} f(x) = 3.5$

LHS = RHS $\lim_{x\to -2} f(x) = 3.5$

$\boxed{f(-2) = 0}$

(4) $\lim_{x\to -1} f(x)$

$\boxed{f(0) = 2}$

(5) $\boxed{\lim_{x\to 0} f(x) = -2}$

(6) $\lim_{x\to 4} f(x)$ LHS: $\lim_{x\to 4^-} f(x) = -2$ RHS: $\lim_{x\to 4^+} f(x) = +\infty$

limit @ $x\to 4$ DNE

$\boxed{f(4) = 0}$

(7) $\lim_{x\to 7} f(x) = 0$

(8) Find $f(-5), f(-2), f(-1), f(0), f(4)$.

(4) $\lim_{x\to -1} f(x)$ limit DNE

LHS $\lim_{x\to -1^-} f(x) = 0 \neq$ RHS $\lim_{x\to -1^+} f(x) = -2$

$f(-1) = -2 = \lim_{x\to -1^+} f(x)$

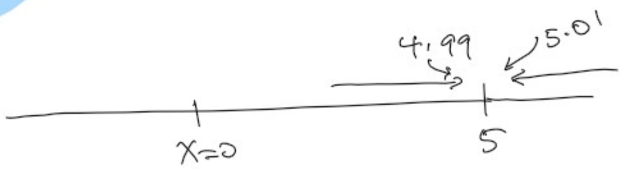
Problem 2. Find the following limits numerically. If a limit does not exist, state this and use the limit notation to describe any infinite behavior.

(1) $\lim\limits_{x\to 1} \dfrac{x^2-1}{x-1}$ $= \underbrace{\dfrac{1^2-1}{1-1} = \dfrac{0}{0}}$ indeterminate

x	0.5	0.9	0.99	1	1.01	1.1	1.5
$f(x)$	1.5	1.9	1.99	$\frac{0}{0}$	2.01	2.1	2.5

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$$

(2) $\lim\limits_{x\to 5} \dfrac{5}{x-5}$



Problem 3. Find $\lim\limits_{x\to 1} \dfrac{x^2-1}{x-1}$ algebraically.

factor

$x^2 - a^2 = (x+a)(x-a)$

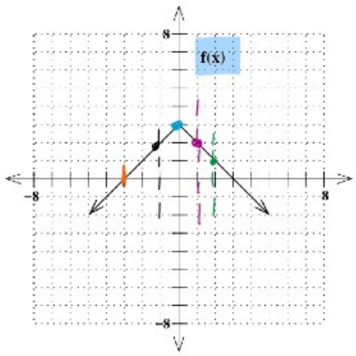
$x^2 - 1 = (x+1)(x-1)$

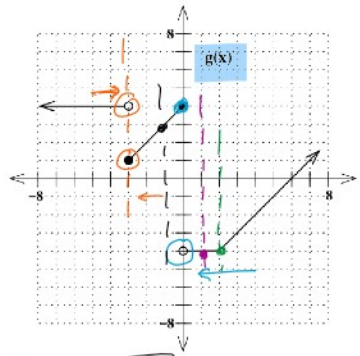
$$\lim_{x\to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$= \lim_{x\to 1} (x+1) = 1+1 = 2$$

Problem 4. Find the following limits, if they exist, based on the graph of $f(x)$ and $g(x)$ below:

$f(-1) = 2$





$g(-1) = 3$

$\lim_{x\to 0} f(x) = 3$

$\lim_{x\to 0} g(x)$ $\to$ -4, 4

(1) $\lim_{x\to 1}[f(x) + g(x)] =$ $2 + (-4) = \boxed{-2}$

(2) $\lim_{x\to 2}[f(x)g(x)] =$ $(1)(-4) = \boxed{-4}$

(3) $\lim_{x\to 0}\left[\frac{f(x)}{g(x)}\right] =$ $\frac{3}{4}$ LHS. $\leftarrow$ check first that $g(x) \neq 0$

RHS $\lim_{x\to 0^+} \frac{f(x)}{g(x)} = \frac{3}{-4} = -\frac{3}{4}$ = RHS.

LHS $\neq$ RHS

$\boxed{\text{Ans: DNE}}$

(4) $\lim_{x\to -3}[x^2 g(x)] =$

LHS: $(-3)^2 \cdot (4)$ RHS: $(-3)^2(1)$

$36 \neq 9$

$\boxed{\text{Ans: } \lim_{x\to -3}[x^2 g(x)] \text{ DNE}}$

$\lim_{x\to -3^-} g(x)$

(5) $\lim_{x\to -1}\sqrt{2f(x) + 4g(x)} =$

$= \sqrt{2(2) + 4(3)}$

$= \sqrt{4+12} = \sqrt{16} = 4$ Ans.

Problem 5. Find the following limits algebraically. If a limit does not exist, state this and use the limit notation to describe any infinite behavior.

(1) $\lim_{x\to 1} \dfrac{x^2 - 3x + 5}{x - 3}$ $= \dfrac{(1)^2 - 3(1) + 5}{(1) - 3} = \dfrac{1 - 3 + 5}{-2} = \dfrac{3}{-2}$ Ans.

$ax^2 + bx + c$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\longrightarrow x = -3, 1$

factor?

(2) $\lim_{x\to 1} \dfrac{x^2 + 2x - 3}{x - 1}$ $= \dfrac{(1)^2 + 2(1) - 3}{1 - 1} = \dfrac{1 + 2 - 3}{0} = \dfrac{0}{0}$ do something more!

$\lim_{x\to 1} \dfrac{(x+3)\cancel{(x-1)}}{\cancel{(x-1)}} = \lim_{x\to 1} (x+3) = 1 + 3 = 4$ Ans.

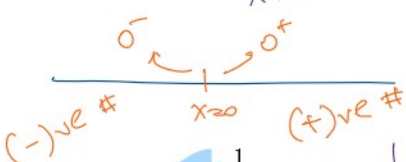
(3) $\lim_{x\to 0} \dfrac{1}{x}$ $= \dfrac{1}{0}$ $\leftarrow$ division by zero NOT ok!

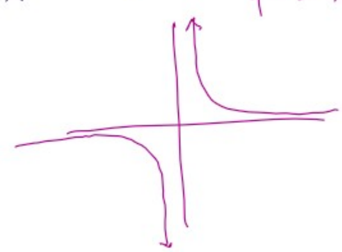
LHS: $\lim_{x\to 0^-} \dfrac{1}{x} = \dfrac{1}{0^-} = -\infty$ | RHS: $\lim_{x\to 0^+} \dfrac{1}{x} = \dfrac{1}{0^+} = +\infty$

$f(x) = \frac{1}{x}$

LHS $\neq$ RHS

limit DNE



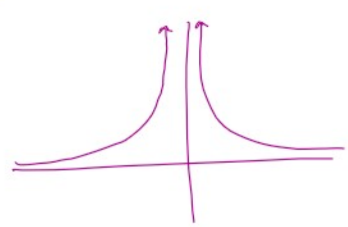


(4) $\lim_{x\to 0} \dfrac{1}{x^2}$ $= \dfrac{1}{0^2}$ $\leftarrow$ division by zero

LHS: $\lim_{x\to 0^-} \dfrac{1}{x^2} = \dfrac{1}{(0^-)^2} = \dfrac{1}{0^+} = +\infty$

RHS: $\lim_{x\to 0^+} \dfrac{1}{x^2} = \dfrac{1}{(0^+)^2} = +\infty$

$\lim_{x\to 0} \dfrac{1}{x^2} = +\infty$



$(-\#)^2 = (+)$ve #

$\frac{0}{0} \rightarrow$ Indeterminate | $\frac{\#}{0} \rightarrow \pm\infty$ | $\frac{0}{\#} = 0$

$\boxed{a^2 - b^2 = (a+b)(a-b)}$

$$\frac{(4)^2 - 4 - 12}{(4)^2 - 16} = \frac{16 - 4 - 12}{16 - 16} = \frac{0}{0}$$

Problem 6. Find $\lim_{x \to 4} \frac{x^2 - x - 12}{x^2 - 16}$ algebraically. If the limit does not exist, state this and use the limit notation to describe any infinite behavior.

$$\lim_{x \to 4} \frac{\cancel{(x-4)}(x+3)}{(x+4)\cancel{(x-4)}} = \lim_{x \to 4} \frac{(x+3)}{(x+4)} = \frac{7}{8} \text{ Ans.}$$

$(x-4)$ term cancelled.

$x - 4 = 0 \Rightarrow x = 4 \leftarrow$ a hole in the graph

vertical asymptote in graph?

If $x + 4 = 0 \rightarrow x = -4$

$$\frac{\left(\frac{1}{5} - \frac{1}{5}\right)}{(5-5)} = \frac{0}{0}$$

Problem 7. Find $\lim_{x \to 5} \frac{\frac{1}{5} - \frac{1}{x}}{5 - x}$ algebraically. If the limit does not exist, state this and use the limit notation to describe any infinite behavior.

$$\lim_{x \to 5} \frac{\left(\frac{x}{x}\frac{1}{5} - \frac{1}{x}\cdot\frac{5}{5}\right)}{5 - x} = \frac{\left(\frac{x-5}{5x}\right)}{5 - x} = \frac{(x-5)}{(5x)(5-x)}$$

$\boxed{\begin{array}{l} x - 5 = -(5 - x) \\ 5 - x = -(x - 5) \end{array}}$

$$\lim_{x \to 5} \frac{\cancel{(x-5)}^{1}}{-\cancel{(x-5)}\cdot(5x)}$$

$$= -\frac{1}{5x} = -\frac{1}{5(5)} = -\frac{1}{25} \text{ Ans.}$$

$$\frac{-1}{5} = \frac{1}{-5} = -\frac{1}{5}$$

for a radical $\rightarrow$ use a conjugate

$$\frac{\sqrt{1+3}-2}{1-1} = \frac{\sqrt{4}-2}{0} = \frac{2-2}{0} = \frac{0}{0}$$

Problem 8. Find $\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1}$ algebraically. If the limit does not exist, state this and use the limit notation to describe any infinite behavior.

$$\sqrt{x+3}-2 \xrightarrow{\text{conjugate}} \sqrt{x+3}+2$$

$$\lim_{x\to 1}\left(\frac{\sqrt{x+3}-2}{x-1}\right)\left(\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}\right)$$

Top: $(a+b)(a-b) = a^2-b^2$

$$\frac{(\sqrt{x+3})^2-(2)^2}{(x-1)(\sqrt{x+3}+2)} = \frac{(x+3)-4}{(x-1)(\sqrt{x+3}+2)} = \frac{\cancel{x-1}}{\cancel{(x-1)}(\sqrt{x+3}+2)}$$

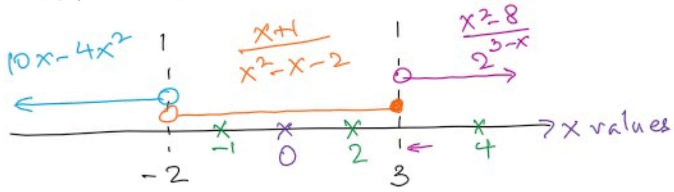
$$= \lim_{x\to 1}\frac{1}{\sqrt{x+3}+2} = \frac{1}{\sqrt{1+3}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$$

Ans.

Problem 9. Find $\lim_{x\to 3} \frac{|x-3|}{6-2x}$ algebraically. If the limit does not exist, state this and use the limit notation to describe any infinite behavior.

Problem 10. Consider the piecewise function $f(x)$ given below, and answer the questions.

$$f(x) = \begin{cases} ① \; 10x - 4x^2 & x < -2 \\ ② \; \dfrac{x+1}{x^2 - x - 2} & -2 < x \leq 3 \\ ③ \; \dfrac{x^2 - 8}{2^{3-x}} & x > 3 \end{cases}$$



$f(3) = \dfrac{3+1}{9-3-2} = \dfrac{4}{4} = 1$

(1) $f(-2) =$ DNE

piece ①

(2) $\lim\limits_{x \to 2} f(x) =$ $10(-2) - 4(-2)^2 = -20 - 16 = -36$

piece ②

(3) $\lim\limits_{x \to -2^-} f(x) =$ $\dfrac{(-2)+1}{(-2)^2 - (-2) - 2} = \dfrac{-1}{4+2-2} = \dfrac{-1}{4}$

(4) $\lim\limits_{x \to -2} f(x) =$ DNE because LHS $\neq$ RHS
part (2) part (3)

(5) $\lim\limits_{x \to 0} f(x) =$ piece ② of $f(x)$

$\dfrac{x+1}{x^2 - x - 2} = \dfrac{0+1}{0-0-2} = \dfrac{1}{-2}$ Ans.

(6) $f(3) =$ 1

$f(x)$ is continuous at $x=3$

LHS = RHS = $f(3)$

LHS

(7) $\lim_{x\to 3^-} f(x) =$ 1

RHS

(8) $\lim_{x\to 3^+} f(x) =$ $\frac{x^2-8}{2^{3-x}} = \frac{3^2-8}{2^{3-3}} = \frac{9-8}{2^0} = \frac{1}{1} = 1$ Ans.

(9) $\lim_{x\to 3} f(x) =$ 1 as LHS = RHS.

$\frac{x+1}{x^2-x-2}$

$\lim_{x\to(-1)} \frac{\cancel{x+1}}{(x-2)(\cancel{x+1})} = \frac{1}{(-1-2)} = -\frac{1}{3}$ Ans

(10) $\lim_{x\to -1} f(x) =$ piece 2 $\frac{(-1)+1}{(-1)^2-(-1)-2} = \frac{0}{1+1-2} = \frac{0}{0}$

(11) $\lim_{x\to 2} f(x) =$ piece 2 $\frac{0}{0} \Rightarrow \lim_{x\to 2} \cdot \frac{(\cancel{x+1})}{(x-2)(\cancel{x+1})} = \frac{1}{2-2} = \frac{1}{0}$

LHS $\frac{1}{0^-} \to -\infty \neq$ RHS $\frac{1}{0^+} \to +\infty$

$\therefore$ limit DNE Ans.

(12) $\lim_{x\to 4} f(x) =$

piece 3 $\frac{x^2-8}{2^{3-x}}$

$\frac{(4)^2-8}{2^{3-4}} = \frac{16-8}{2^{-1}} = \frac{8}{(2^{-1})} = (8)(2) = 16$ Ans.

$\frac{1}{2^{-1}} = 2^1 = 2$